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An Empirical Investigation of the Variance Estimation Methodology Prescribed for the National Stream Survey: Simulated Sampling from Stream Data Sets

Stephen V. Stehman
W. Scott Overton

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by Stephen V. Stehman
and W. Scott Overton

Biometrics Unit, Cornell University
and Department of Statistics, Oregon State University

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1.0 Introduction

These simulation studies were conducted to validate the use of prescribed methodology (Overton, 1985, 1987) in the National Stream Survey (NSS). Throughout the NSS, population quantities of interest were estimated using the Horvitz-Thompson estimator, $\hat{T}_y$. Two aspects of the prescribed method for estimating the variance of $\hat{T}_y$ required confirmation. First, although the sample was taken in a systematic manner from the topographic maps, the variance was estimated as though the list of reaches had been randomly sorted prior to selection of the sample. Second, an approximate formula to the pairwise inclusion probabilities was used in calculating variances.

1.1 Design and Estimation in the NSS

The Phase I Stream Survey design was a variable probability, systematic (hereafter denoted *vps*) sample (Overton, 1985). Sample elements were selected using a point/area sampling frame imposed on topographic maps of the target area. Each point in the square dot grid was associated with a target reach or "no reach", where a reach was a well-defined stream segment. This protocol resulted in reaches being sampled with probability proportional to direct watershed area.

The Horvitz-Thompson Theorem of probability sampling provided the basic estimation strategy for the survey. For each element in a finite population of size N, let y_i be the variable of interest, and x_i ($x_i > 0$) be an auxiliary variable used to select the sample, s (in the NSS, x_i = direct watershed area). Then the sampling design determines the inclusion probabilities, $\pi_i = \Pr\{i \in s\}$. The Horvitz-Thompson estimator, $\hat{T}_y = \sum_{i \in s} \frac{y_i}{\pi_i}$, is unbiased for the population total, $T_y = \sum_{i=1}^{N} y_i$, and has variance

$$V(\hat{T}_y) = \sum_{i=1}^{N} \left(\frac{y_i}{\pi_i}\right)^2 (1-\pi_i)\,\pi_i + \sum_{i=1}^{N}\sum_{j\neq i}^{N} \left(\pi_{ij} - \pi_i\pi_j\right) \frac{y_i}{\pi_i}\frac{y_j}{\pi_j} \tag{1.1}$$

$$= \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left(\pi_i\pi_j - \pi_{ij}\right) \left(\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j}\right)^2 \tag{1.2}$$

where π_{ij} is the probability that units i and j are both selected in the sample (the pairwise inclusion probability). Equation (1.1) holds in general, (1.2) only if the sample size is fixed.

Horvitz and Thompson (1952) also provided an estimator for $V(\hat{T}_y)$,

$$v_{HT} = \sum_{i=1}^{n} \left(\frac{y_i}{\pi_i}\right)^2 (1-\pi_i) + \sum_{i=1}^{n}\sum_{j\neq i}^{n} \left[\frac{\pi_{ij} - \pi_i\pi_j}{\pi_{ij}}\right] \frac{y_i}{\pi_i}\frac{y_j}{\pi_j} , \tag{1.3}$$

where i and j are now used as indices on the sample. v_{HT} is the form prescribed in the Stream Survey. We also examined an alternative variance estimator (Yates and Grundy, 1953) that has gained favor in the literature:

$$v_{YG} = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left[\frac{\pi_i\pi_j - \pi_{ij}}{\pi_{ij}}\right] \left[\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j}\right]^2 . \tag{1.4}$$

Both v_{YG} and v_{HT} are unbiased if $\pi_{ij}>0$ for all population pairs i and j.

A general asssment of these two estimators is made in Stehman and Overton (1987a). The original reason for choice of 1.3 was simplicity of the recursive form, along with the conceptual subpopulation identity. As we worked with the two forms, certain of the initial perceptions of difference between 1.3 and 1.4 disappeared, but others arose. The resultant view is that 1.3 is the more appropriate for the stream survey, and the contrast with the behavior of 1.4 made in this report is more for perspective than contrast with a true alternate method.

1.2 Estimating the Variance for Systematic Samples

The systematic sampling procedure of the NSS will be termed "fixed configuration" *vps* sampling. The sample was selected systematically from the population in its given fixed, spatial arrangement. The population units were not randomly ordered prior to selection of the sample. "Random-order" *vps* sampling will refer to a systematic sampling procedure in which the list of population elements is randomly permuted prior to selecting the sample. The random order systematic variable probability sample has been used for many years; the Hartley and Rao (1962) reference is an important link to early treatment of this sampling design.

Estimating the variance for a fixed configuration, systematic sample poses several problems. Exact determination of the π_{ij}'s in either fixed configuration or random-order systematic sampling is computationally difficult and requires knowledge of all x_i's in the population. In fixed configuration systematic sampling, some π_{ij}'s are likely to be very small, and many of the π_{ij}'s are zero.

It is clear that instability of the variance estimator is caused by very small π_{ij}'s, and that the fixed configuration model is characterized by extremely small π_{ij}'s, in addition to many zero π_{ij}'s. Recall that the theoretical unbiasedness of the estimated variance, whether by v_{YG} or v_{HT}, depends on all the π_{ij}'s being non-zero. The nature of the bias that accompanies any sampling scheme having a substantial number of π_{ij}'s identically zero is given by rewriting Equation 1.1, with the second term defined on each of the two sets of (i,j)-pairs:

$$V(\hat{T}_y) = \sum_i \left[\frac{y_i}{\pi_i}\right]^2 (1-\pi_i)\pi_i + \sum\sum_{i,j\neq i:\pi_{ij}\neq 0} \left[\pi_{ij}-\pi_i\pi_j\right]\frac{y_i y_j}{\pi_i\pi_j} - \sum\sum_{i,j\neq i:\pi_{ij}=0} y_i y_j$$

$$= A + B - C.$$

Then v_{HT} is unbiased for A + B, and the bias of v_{HT} is easily identified as the quantity,

$$\text{BIAS} = C = \sum\sum_{i,j\neq i:\pi_{ij}=0} y_i y_j \;.$$

This bias may be considerable in fixed configuration sampling because of the substantial number of zero π_{ij}'s that occur. Zero π_{ij}'s do not occur under the randomization model and in that model the only very small π_{ij}'s are associated with extremely small (π_i, π_j) pairs. In contrast, in fixed configuration systematic sampling, very small π_{ij}'s may be associated with the largest π_i's, so that much better behavior of v_{HT} is anticipated for the randomization model than for the fixed configuration model. Similar behavior is anticipated for v_{YG}, but its bias has not been assessed.

To get around these difficulties with fixed configuration variance estimation, the approach to variance estimation prescribed for the NSS was to treat the fixed configuration systematic sample as a random-order, systematic sample; that is, the observed configuration was treated as though it were a realization of a random process. The variance estimation model was that appropriate for a randomly ordered population. This approach has a precedent in the analysis of a common (equal probability) systematic sample. Variance estimates for the common systematic sample are often computed using the formula appropriate for a simple random sample (cf. Cochran, 1977). Since a simple random sample is the uniform probability analog of a random order, variable probability sample, we extended the same variance estimation model to the variable probability case.

For many natural configurations, it is reasonable to treat the observed configuration as a random one. Several authors have demonstrated that the consequence of this approach to variance estimation in natural populations is usually slight overestimation of the variance. This is essentially a statement about the natural configurations that occur in those circumstances, and the correlation patterns that exist in the dimension of systematic sampling. Milne (1959) studied the appropriateness of the random model of variance estimation for uniform probability systematic samples from many natural populations. He concluded that generally one would not go far wrong treating a centric, systematic area-sample as if it were random. But essentially this is a point that must be established for any circumstance of application.

In the common systematic sample, it has long been recognized that if a systematic effect is present, so that the estimator is more precise than a simple random sample, then the variance estimated by the simple random sample formula will overestimate the actual variance. Similarly, if the systematic effect leads to increased variance, this variance will be underestimated. We anticipate this see-saw systematic effect to carry over into *vps* sampling.

More sophisticated variance estimation models can reduce the problem of overestimation. Overton (1964) explored variance estimation for common systematic sampling, and recommended the mean square successive difference estimator. Wolter (1985) listed several alternative variance estimators applicable for both uniform and *vps* sampling, each based on an hypothesized model for the underlying population. While it is worthwhile to consider variance estimators designed for particular underlying models of population structure, we first explored the general application of the randomized configuration variance estimator. If most natural population configurations can be treated as random, at least for the variable probability case, there is no need to employ more complicated variance estimators. Further, most of the proposed alternative variance estimators for variable probability sampling are somewhat ad hoc in their treatment of the finite nature of the population, either ignoring the without replacement aspect of the sampling design, or constructing an approximate finite population correction factor. This correction is built into the Horvitz-Thompson variance estimator as well as the Yates-Grundy form.

The Hartley and Rao (1962) approximate π_{ij} formula was proposed to be used for populations that could be considered in random order. Wolter used this π_{ij} approximation with the Yates-Grundy estimator for this circumstance, but did not suggest the Horvitz-Thompson variance estimator. Isaki and Pinciaro (1977) empirically compared these alternative estimators for fixed configuration, *vps* sampling. Based on two populations and two distinct orderings of these populations, their general conclusion was that the choice of variance estimator depended on the relationship between the ordering variable and the ratios y/x. For certain relationships, treating the sample as random led to overestimation of variance. Our investigation extends these studies in the context of the stream population.

1.3 Pairwise Inclusion Probability Formulae

The variance estimation model required computing the pairwise inclusion probabilities under the assumption of *vps* sampling from a randomly ordered list. Formulas for the exact π_{ij}'s were not usable in the stream survey because all x_i's in the population were not known. The Hartley and Rao (1962) approximate π_{ij} formula has been extensively studied in the statistical literature. This formula also requires knowledge of all population π_i's and therefore was not available for use in the stream survey.

An approximation to the π_{ij}'s allowing simple formulae and computing algorithms for variance estimates, and requiring knowledge only of x's in the sample, was prescribed by Overton (1985):

$$\pi_{ij}^{o} = \left[\frac{\pi_i(n-1)x_j}{\left(T_x - \frac{x_i}{2} - \frac{x_j}{2}\right)} \right], \tag{1.5}$$

where T_x is the population total of the x's.

Even though the Hartley-Rao approximation was not usable, it was of interest to study comparative performance of the variance estimators using π_{ij}^{o} and the Hartley-Rao formula in the context of stream populations, under random-order, *vps* sampling (Stehman and Overton, 1987).

1.4 Selection of a Variance Estimator in the NSS

The variance estimator prescribed for the NSS, here denoted by v^{o}_{HT}, was v_{HT} calculated using π^{o}_{ij}. This formula met the needs of computational convenience and general utility, and also provided a means for incorporating the randomized population assumption. Because of the poor results that have been reported for v_{HT} in the literature, and because of our interest in more adequately dealing with variance estimation in the fixed configuration case, the prescribed variance estimator was subjected to a number of comparisons of behavior. Additionally, we explored formulae of variations that seemed promising in the fixed configuration case (Sec. 1.5).

Although we have compared the performance of the Horvitz-Thompson variance estimator to the performance of the Yates-Grundy formula, the latter is not really a choice for many of the estimates produced in the Stream Survey. The original reason for using the Horvitz-Thompson variance was the ready adaptation to a recursive form. We have since also generated a recursive Yates-Grundy form, so that reason no longer exists. However, in exploration of the two forms, it was discovered that the Yates-Grundy form cannot be used to represent unconditional variance in certain sub-populations. Specifically, in the Stream Survey, the number of sample target reaches is a random variable, and the appropriate structure does not exist to construct the Yates-Grundy formula on the dot-grid points that do not lead to a reach.

It is important to note that estimation of T_y is not an issue. $\hat{T}_y$ is unbiased whether the systematic sampling is random-order or fixed configuration. However, the variance of $\hat{T}_y$ can be different in the two circumstances, and it is the difference in variance estimates that is of interest in this study.

1.5 Modified Estimators of Variance

It is anticipated that the variance estimators, v_{HT} and v_{YG}, will often give conservative estimates of variance, due to the systematic effect. A naturally considered estimator that should reflect any gain in precision achieved by fixed configuration over random-order systematic sampling is an analog of the mean-square successive difference, δ^2. This statistic, due to von Neuman et al (1941), has been shown to adequately express error variance of a common systematic sample

(Overton, 1964). In the usual formula for variance, the sample variance, s^2, is replaced by δ^2, where

$$\delta^2 = \frac{1}{2(n-1)} \sum_{i=1}^{n-1} \left[y_i - y_{i+1}\right]^2. \tag{1.6}$$

The form of this is strongly suggestive of the Yates-Grundy variance estimator. Equation 1.4 can be rewritten as the linear function of a set of difference estimators, each having the form of 1.6, with different order lag:

$$v_{YG} = \sum_{j=1}^{n-1} a_j v_{YG}^{sj}, \tag{1.7}$$

where,

$$v_{YG}^{sj} = b_j \sum_{i=1}^{n-j} c_{ij} d_{ij}^2,$$

$$c_{ij} = (\pi_i \pi_{i+j} - \pi_{i,i+j})/\pi_{i,i+j}, \text{ and}$$

$$d_{ij} = \left[\frac{y_i}{\pi_i} - \frac{y_{i+j}}{\pi_{i+j}}\right].$$

In this formulation, $a_j = b_j^{-1}$.

Consider the first order successive differences, $d_{i1} = \left(\frac{y_i}{\pi_i} - \frac{y_{i+1}}{\pi_{i+1}}\right)$. The resultant formula for the first order variance estimator v_{YG}^{s1} is:

$$v_{YG}^{s1} = \frac{n}{2} \sum_{i=1}^{n-1} \left(\frac{\pi_i \pi_{i+1}}{\pi_{i,i+1}} - 1\right) \left(\frac{y_i}{\pi_i} - \frac{y_{i+1}}{\pi_{i+1}}\right)^2. \tag{1.8}$$

Choice of the value of b_1 in (1.8) follows this reasoning: the usual Yates-Grundy estimator is a sum of $n(n-1)/2$ terms, while there are only $n-1$ differences summed in v_{YG}^{s1}. The multiplier $n/2$ in v_{YG}^{s1} scales the estimator to be equivalent to a sum over $n(n-1)/2$ terms. If sampling is with uniform inclusion probabilities, (1.8) is equal to (1.6).

A second variance estimator, based on the second order successive differences, was also examined. The specific formula for this estimator is:

$$v_{YG}^{s2} = \frac{n(n-1)}{2(n-2)} \sum_{i=1}^{n-2} \left(\frac{\pi_i \pi_{i+2}}{\pi_{i,i+2}} - 1\right) \left(\frac{y_i}{\pi_i} - \frac{y_{i+2}}{\pi_{i+2}}\right)^2. \tag{1.9}$$

Again, the value, $b_2 = n(n-1)/2(n-2)$, was chosen to make the estimator equivalent to a sum over $n(n-1)/2$ squared differences.

2.0 Procedure for Confirmation of NSS Variance Equations

The simulation studies were designed to confirm the utility of the approximation formulas for the π_{ij}'s for random-order, *vps* sampling, then to examine the randomization assumption for fixed configuration sampling from stream population data. The GAUSS Statistical Software package (Version 1.49, Aptech Systems, Kent, WA) was used for all simulations.

The first set of simulations (Table 1) verified the simulation algorithm and the computing formulae by examining the variance estimators for a simple random sample. This was followed by simulations using populations from the statistical literature, comparing the variance estimators computed using π_{ij}^{o} (Table 2). These simulations assess the adequacy of the variance estimators for random-order, *vps* sampling.

The next set of simulations (Tables 3a, 3b and 3c) uses a data set, SS2, from the pilot survey of the NSS (data in Appendix A). These simulations were designed to study the properties of the variance estimators for both random-order and fixed configuration *vps* sampling from a population similar in structure to those encountered in the NSS. Fixed configuration sampling was carried out on the population in its original (natural) order, and also ordered by x, y, and the ratio y/x. Ordering on the x's is a feasible design consideration when all the x's are known, while ordering on the y's is the most favorable configuration, in terms of precision of estimator, for equal probability, systematic sampling. Ordering on y/x is a very favorable configuration for *vps* sampling.

The rows in Table 3 under "restricted randomization" were simulations designed to explore another model of population structure, one we will term "local randomization". This model, a compromise between the random-order and fixed configuration models, treats a specific configuration as representative, to some extent, of the underlying population structure, but also permits some degree of randomness within that structure. That is, overall patterns in the population are considered meaningful, but some degree of "exchangeability" of population units is possible locally.

The methods for making this model operational for simulation studies will clarify the idea. Two methods of restricted randomization were studied. In one, the set of 100 population units was divided into subgroups of 10 each, and the units within each subgroup were randomly ordered. Thus a unit could not change subgroups, maintaining overall population patterns, but a unit could change positions within the subgroup, providing local randomization. The other restricted randomization added a uniformly distributed error term to the ratio y/x, and re-ordered the population on this new variable. These new ratios were used only to re-order the population; the original ratios were used in the variance calculations. This method was used starting with the population ordered on the ratios y/x.

Two indices were calculated to measure the degree of change realized by the methods of re-ordering the populations. If i is the original position of a unit in the population list, and j is the position after re-ordering, one index calculated was the mean absolute difference, $\sum_{i=1}^{N}\sum_{j>i}^{N}|i-j|/N(N-1)$, while the other index was the mean squared difference, $(i-j)^2$. The values obtained were:

	Mean Abs Diff	Mean Square Diff
Error: U[-.10, .10]	4.35	32
Error: U[-.15, .15]	6.25	65
Subgroup randomization	3.25	16

The final group of simulations (Tables 4 and 5) studied the properties of modified variance estimators described in Section 1.5. Simulations were carried out on stream population SS2 for samples of size 8 and 16, and on another stream population, KNOX, for samples of size 75.

3.0 Results

Notation used in Tables

T_y = population total

$\hat{T}_y$ = Horvitz-Thompson estimator of the population total. The reported value is the mean over all replications. (Note that the expected value of $\hat{T}_y$ is T_y.)

$V(\hat{T}_y)$ = true variance of $\hat{T}_y$.

v^o_{HT} = Horvitz-Thompson variance estimator of $V(\hat{T}_y)$ calculated using π^o_{ij}. The reported value is the mean over all replications.

v^{o}_{YG} = Yates-Grundy variance estimator of $V(\hat{T}_y)$ calculated using π^{o}_{ij}. The reported value is the mean over all replications.

$\hat{V}(\hat{T}_y)$ = unbiased estimator of $V(\hat{T}_y)$. $\hat{V}(\hat{T}_y)$ is the sample variance of $\hat{T}_y$ over all replications in the simulation.

$s(v^{o}_{HT})$ and $s(v^{o}_{YG})$ = standard deviation of v^{o}_{HT} or v^{o}_{YG} calculated as the square root of the sample variance of v^{o}_{HT} or v^{o}_{YG} over all replications in the simulation.

rel bias $(v^{o}_{HT}) = \left[v^{o}_{HT} - \hat{V}(\hat{T}_y)\right] / \hat{V}(\hat{T}_y)$

List of Populations Studied

R Raj (1965) and Horvitz and Thompson (1952)
(x=eye estimate of number of households, y=number of households)

RM Raj (1965)
(same population as R with two elements modified to decrease relationship between x and y)

L Levy and Lemeshow (1980)
(x=average number of hospital admissions per day, y=number of beds)

K Kish (1965)
(x=number of dwellings, y=number of renters in a block)

SS2 National Stream Survey - first 100 pilot study reaches
(x=direct watershed area, y=reach length)

KNOX Overton and Stehman (1987)
All target reaches of the Knoxville quadrangle of the National Stream Survey
(x=direct watershed area, y=reach length)

3.1 Studies to Establish the Validity of the Simulation Algorithm

The validity of the simulation algorithm was established by comparing simulation results with known values. The results for simple random sampling from Population R are given in Table 1. The two variance estimators, v_{HT} and v_{YG}, are algebraically identical under simple random sampling, and the simulations gave identical numerical values. In this table, it seems that all values are within the expected range, as assessed by use of probable error calculations: P.E. = 0.6745*SE.

Other calculations:

$$SE(\hat{T}_y) = \sqrt{V(\hat{T}y)/100} = \sqrt{16,221/100}$$

$$SE(v_{HT}) = \sqrt{V(v_{HT})/10} = 26,539/\sqrt{10}$$

$$SE[\hat{V}(\hat{T}_y)] = s[\hat{V}(\hat{T}_y)]$$

Table 1. Simple Random Sample Simulations, Population R

(Each rep is the mean of 100 samples of size n=2)

Rep.	$\hat{T}_y$	v_{HT} (or v_{YG})	$\hat{V}(\hat{T}_y)$	$s(v_{HT})$
1	425	16,262	16,957	24,973
2	420	15,314	14,646	21,562
3	425	14,879	16,905	21,555
4	432	15,135	14,680	20,661
5	450	13,627	17,784	18,437
6	454	17,577	19,454	24,476
7	441	15,653	18,042	22,963
8	441	16,204	17,456	24,236
9	438	13,742	12,967	23,369
10	446	16,176	13,924	25,575
Mean	437	15,457	16,281	22,878[b]
Stnd Dev	11.4	1,202	2,093	
Expected	434	16,221	16,221	26,539[a]

[a] Value reported by Raj (1965); [b] square root of mean s^2

Results:

$T_y \pm$P.E. = [425.4, 442.6], <3,4,3>

$E(v_{HT}) \pm$P.E. = [14,430, 18,012], <2,8,0>

$E[\hat{V}(\hat{T}_y)] \pm$P.E. = [14,809, 17,633], <4,3,3>.

The numbers in < > are compared against the expectation of a value being below the probable error, within the probable error, or above the probable error, <1/4, 1/2, 1/4>, which for 10 replications becomes <2.5, 5.0, 2.5>. A Chi-square test for goodness-of-fit is calculated for each of these variables, yielding:

$$\chi^2 = 0.4 \text{ for } \hat{T}_y; \quad \chi^2 = 4.4 \text{ for } v_{HT} \text{ and } v_{YG}; \quad \chi^2 = 0.7 \text{ for } \hat{V}(\hat{T}_y).$$

These Chi-square values are well below the critical value of 6.0.

3.2 Evaluation of Variance Estimators using π^o_{ij} Under Random Order Sampling

Evaluation of the π^o_{ij} approximation formula was made in the context of several populations taken from the statistical literature, Table 2.

Table 2. Evaluation of Variance Estimators: Literature Populations (Random-order, *vps* sampling. 1,000 reps)

Pop.	n	T_y	$\hat{T}_y$	v^o_{HT}	v^o_{YG}	$\hat{V}(\hat{T}_y)$	$s(v^o_{HT})$	$s(v^o_{YG})$
R	2	434	431	2,920	2,877	3,085	3,929	4,001
RM	2	434	434	8,783	8,356	8,541	18,502	18,040
L	2	264	264	2,301	2,215	2,285	5,405	5,143
R	4	434	433	1,352	1,314	1,244	908	962
RM	4	434	433	4,426	3,914	3,842	5,617	5,262
K	2	2, 810	2, 802	95,992	103,400	108,400	148,100	152,900
K	4	2, 810	2, 802	47,523	51,409	54,355	45,878	48,143
K	8	2, 810	2, 809	19,687	24,106	23,311	14,412	16,193

These results confirm that the variance estimators, incorporating π^o_{ij}, work well for the case when the population is randomly ordered prior to selection of the sample. The properties of v^o_{HT} and v^o_{YG} were very similar. Bias was small for both variance estimators, except for v^o_{HT} in population K.

3.3. Evaluation of Variance Estimators Using Stream Survey Data, Population SS2

Table 3a. Properties of Variance Estimators Under Various Population Orderings (n=8; 5,000 reps, T_y = 432.6)

	$\hat{T}_y$	v^o_{HT}	v^o_{YG}	$\hat{V}(\hat{T}_y)$	$s(v^o_{HT})$	$s(v^o_{YG})$
Random-order	432.9	5,577	4,851	4,924	15,397	13,918
Fixed Configuration						
Natural order	433.1	6,026	5,228	2,538	16,138	14,624
Order on x	432.8	5,725	4,983	4,733	16,673	15,407
Order on y	433.2	5,434	4,694	6,455	14,465	13,180
Order on y/x	432.7	5,835	5,125	2,117	13,995	12,937
Restricted Randomization						
Natural order (subgroups)	433.9	5,766	5,020	4,342	15,474	14,105
Order on y/x (subgroups)	432.5	5,892	5,124	2,312	14,571	13,262
Order on y/x (error 1[a])	433.4	6,152	5,355	2,067	15,523	14,117
Order on y/x (error 2[b])	432.5	5,812	5,052	2,437	14,173	12,861

[a] error 1 is U[-.10, .10]   [b] error 2 is U[-.15, .15]

Table 3b. Properties of Variance Estimators Under Various Population Orderings (n=16; 5,000 reps, T_y = 432.6)

	$\hat{T}_y$	v^o_{HT}	v^o_{YG}	$\hat{V}(\hat{T}_y)$	$s(v^o_{HT})$	$s(v^o_{YG})$
Random-order	432.0	2,703	2,007	2,242	4,866	3,988
Fixed Configuration						
Natural order	433.2	2,954	2,179	789	5,429	4,203
Order on x	432.8	2,738	2,027	2,980	5,136	4,314
Order on y	431.9	2,742	2,036	2,533	4,883	4,041
Order on y/x	432.6	2,907	2,159	903	5,236	4,273
Restricted Randomization						
Natural order (subgroups)	432.5	2,811	2,093	1,655	5,127	4,193
Order on y/x (subgroups)	433.0	2,977	2,226	878	5,542	4,560
Order on y/x (error 1[a])	432.1	2,781	2,060	875	4,603	3,760
Order on y/x (error 2[b])	432.6	2,862	2,130	988	5,032	4,115

[a] error 1 is U[-.10, .10] [b] error 2 is U[-.15, .15]

Table 3c. Confidence Interval Coverage and Relative Bias (5,000 reps)

	Confidence Interval Coverage[a]				rel bias			
	n=8		n=16		n=8		n=16	
	v^o_{HT}	v^o_{YG}	v^o_{HT}	v^o_{YG}	v^o_{HT}	v^o_{YG}	v^o_{HT}	v^o_{YG}
Random-order	89	88	93	90	.13	-.02	.21	-.10
Fixed Configuration								
Natural order	99	99	100	100	1.37	1.06	2.74	1.76
Order on x	94	93	90	87	.21	.05	-.08	-.32
Order on y	86	78	96	80	-.16	.27	.08	-.20
Order on y/x	100	100	100	100	1.76	1.42	2.22	1.39

[a] Values reported are the percentage of samples in which $\hat{T}_y \pm 2*\sqrt{\hat{v}}$ covers the true parameter, T_y (nominal 95% intervals).

As was observed in the simulations for random-order sampling from the populations in Table 2, both variance estimators are adequate for random-order, *vps* sampling from the stream data. Better confidence interval coverage was obtained using v^o_{HT}, but v^o_{YG} had smaller bias and variance. The behavior of the variance estimators for fixed configuration, *vps* sampling depended on whether the configuration was a favorable one for the sampling scheme. Looking at the column estimating $V(\hat{T}_y)$, the natural configuration appeared highly favorable for *vps* sampling, $\hat{V}(\hat{T}_y)$ being almost the same as that of ordering on y/x. Ordering on x or y is roughly equivalent to random-order sampling.

The variance estimators appeared not to reflect the changes in $V(\hat{T}_y)$ produced by the different orderings. Since the variance estimators are prescribed for sampling from a randomly ordered list, we would expect the results to exhibit the "see-saw" effect described in Section 1.2. The two orderings showing the greatest decrease in $\hat{V}(\hat{T}_y)$, natural ordering and ordering on the ratios y/x, did result in slightly higher variance estimates relative to the random-order estimates. But generally, the variance estimates more closely reflected the variance under random-order *vps* sampling, and the see-saw effect was not very distinct.

The overall assessment is that v^{o}_{HT} and v^{o}_{YG} provide good estimates of variance for random-order *vps* sampling, and also for fixed configuration *vps* sampling, if the gain in precision from ordering is not too large. When a substantial gain in precision is achieved through the fixed configuration sampling scheme, the variance estimators are very conservative. The alternate estimators are assessed in Section 3.4.

The variance estimators for the restricted randomization simulations showed very little change from the estimators in their fixed configuration counterparts. Similarly little change was seen in $\hat{V}(\hat{T}_y)$. Local randomization does not destroy the gain in precision nor the associated conservatism of the variance estimators deriving from a strong ordering. The conclusion from these simulations is that since the restricted randomization resulted in little change, regarding the populations as locally randomized is reasonable. On the other hand, less restricted randomization must be assumed in order to justify the basic variance estimators under a strong systematic effect.

3.4 Modified Variance Estimators

In Section 1.5, alternate variance estimators were considered for the purpose of reflecting the gain in precision from the systematic effect. The results of using v^{s1}_{YG} and v^{s2}_{YG} to estimate $V(\hat{T}_y)$ for fixed configuration systematic sampling for various orderings of Population SS2 are shown in Tables 4a and 4b.

Table 4a. Comparison of Modified Variance Estimators[a] for Fixed Configuration Sampling from Population SS2. Variable = LENGTH (n=8; 5000 reps)

	v^{o}_{HT}	v^{o}_{YG}	v^{s1}_{YG}	v^{s2}_{YG}	$\hat{V}(\hat{T}_y)$
Natural order	6,026	4,754	3,825	2,558	2,202
Order on x	5,725	5,032	3,285	1,858	4,744
Order on y	5,434	4,529	3,803	2,837	6,250
Order on y/x	5,835	5,234	1,191	1,352	2,210
Random order	4,782	4,146	4,130	4,071	4,388

[a] v^{s1}_{YG} and v^{s2}_{YG} are calculated using π^{o}_{ij}.

Table 4b. Comparison of Modified Variance Estimators[a] for Fixed Configuration Sampling from Population SS2. Variable = LENGTH (n=16, 5000 reps)

	v^{o}_{HT}	v^{o}_{YG}	v^{s1}_{YG}	v^{s2}_{YG}	$\hat{V}(\hat{T}_y)$
Natural order	2,954	2,064	1,596	1,167	680
Order on x	2,738	2,001	1,140	773	2,927
Order on y	2,742	1,965	2,110	1,731	2,393
Order on y/x	2,097	2,090	361	379	834
Random order	2,713	2,007	2,052	2,028	2,242

[a] v^{s1}_{YG} and v^{s2}_{YG} are calculated using π^{o}_{ij}.

Based on these comparisons, use of the modified Yates-Grundy estimators does not appear justified. v^{s2}_{YG} severely underestimates the variance of $\hat{T}_y$ in all but the natural ordering. The underestimation of v^{s1}_{YG} is not as extreme as that of v^{s2}_{YG}, but the negative bias is still large. For the natural ordering, both v^{s1}_{YG} and v^{s2}_{YG} have smaller bias than v^{o}_{YG} and v^{o}_{HT}, but here the four estimators are positively biased. An encouraging property of v^{s1}_{YG} and v^{s2}_{YG} is that both seem to provide nearly unbiased estimates in random-order *vps* sampling. It is interesting to note that while the natural ordering and ordering on y/x result in roughly the same $\hat{V}(\hat{T}_y)$, the structure producing this gain is evidently different. The pattern of differences between v^{s1}_{YG}, v^{s2}_{YG}, and $\hat{V}(\hat{T}_y)$ for these two orderings apparently reflects some difference in population structure.

Table 5a. Fixed Configuration Sampling from Population KNOX[a]
Variable = LENGTH (T_y = 4,192)
(n=75; 1500 reps)

	$\hat{T}_y$	v^o_{HT}	v^o_{YG}	v^{s1}_{YG}	$\hat{V}(\hat{T}_y)$
Natural order	4,188	57,579	51,870	51,609	56,466
Order on x	4,200	68,333	61,972	30,658	28,759
Order on length	4,184	57,634	51,847	48,746	42,160
Order on length/x	4,190	59,734	53,859	5,669	13,214
Random order	4,191	58,426	52,627		55,339

[a] v^{s1}_{YG} is calculated using π^o_{ij}.

Table 5b. Fixed Configuration Sampling from KNOX[a]
Variable = NUMBER OF REACHES (T_y = 1296)
(n=75; 1500 reps)

	$\hat{T}_y$	v^o_{HT}	v^o_{YG}	v^{s1}_{YG}	$\hat{V}(\hat{T}_y)$
Natural order	1,297	42,843	40,045	39,865	35,127
Order on x	1,297	40,843	38,177	10,119	18,524
Order on length/x	1,300	41,938	39,168	19,667	27,461
Random order	1,298	39,738	37,119	----	38,269

[a] v^{s1}_{YG} is calculated using π^o_{ij}.

For the two variables examined in population KNOX, the natural order fixed configuration behaved very similarly to random order. Ordering on the x's resulted in increased precision of $\hat{T}_y$ in both Tables 5a and 5b. The estimator v^{s1}_{YG} had smaller bias than the basic variance estimators v^o_{HT} and v^o_{YG}, but tended to be anti-conservative. For the response variable length, v^{s1}_{YG} provided a good estimate of variance for all orderings except the extreme case of ordering on the ratios y/x. While v^{s1}_{YG} adequately estimated the variance for the natural ordering in Table 5b, it underestimated the variance when the population was ordered on x or length/x. Generally, it appears that v^{s1}_{YG} adequately reflected the fixed configuration variance if the increase in precision of $\hat{T}_y$ due to a favorable ordering was not too great. If the gain in precision was substantial, v^{s1}_{YG} tended to underestimate the variance.

4.0 Summary

For random-order, *vps* sampling, both v^{o}_{HT} and v^{o}_{YG} provide nearly unbiased estimates of variance. Stehman and Overton (1987) present further simulation evidence showing that v^{o}_{HT} usually provides better confidence interval coverage and slightly higher MSE compared to v^{o}_{YG}. Further, both variance estimators computed using π^{o}_{ij} have better properties than the estimators computed using the Hartley-Rao π_{ij} approximation formula.

For fixed configuration sampling, the variance estimators prescribed for the model of a randomly-ordered population can be very conservative. The modified variance estimators based on the mean-square successive difference were potentially useful alternatives in the particular small stream population studied. These modified estimators appeared to work better in the larger stream population, KNOX (Table 5). Other simulation studies (not reported here) examining these modified estimators have shown more promising results, but further work is needed.

Based on the results of this study, the conservative estimator, v^{o}_{HT}, based on the random-order model, and prescribed in the analysis plan (Overton, 1985, 1987), is adequate in assessing variance in the National Stream Survey. This position is reinforced by results of a replicated sample (Overton and Stehman, 1987). Slight gains are apparently possible from use of modified variance estimators that reflect the gain in precision from a favorable ordering, but those assessed require more work before they can be used with any degree of confidence.

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Appendix A: Stream Population SS2

x	y	x	y	x	y	x	y
1.69	1.98	8.01	3.64	13.34	8.90	3.72	3.32
1.55	2.41	7.76	6.16	3.28	3.34	3.64	2.40
2.19	2.84	2.76	1.93	15.08	6.22	36.05	14.30
1.52	1.95	11.75	3.90	2.66	1.84	6.41	0.77
2.54	3.18	15.29	7.08	26.17	9.41	2.22	2.32
3.63	3.57	2.27	2.14	27.35	13.53	9.09	4.49
0.49	2.78	4.66	2.28	5.90	2.82	12.26	4.80
0.99	2.09	6.00	5.91	4.79	2.52	10.92	5.49
8.50	9.12	7.44	5.88	4.67	2.53	1.14	0.66
3.31	4.51	5.02	3.50	2.93	2.96	3.84	3.61
1.68	2.33	8.54	5.39	1.34	1.22	2.54	1.54
0.60	1.23	8.32	5.52	9.59	5.29	24.71	10.98
2.48	2.71	1.97	1.53	9.10	9.10	2.96	3.16
3.21	3.02	10.54	2.98	13.79	6.15	6.58	4.24
3.23	2.83	5.08	5.14	4.81	2.17	7.56	3.88
2.31	3.24	2.36	1.40	16.68	6.99	15.04	6.10
1.96	2.13	7.33	4.55	5.16	3.14	9.05	5.99
1.36	1.83	1.98	1.65	10.91	10.64	12.38	3.97
1.88	2.70	1.87	1.61	11.82	6.76	8.61	6.14
1.29	1.38	15.28	10.36	11.93	5.14	22.73	19.98
0.34	0.58	2.21	3.05	10.35	3.92	3.62	2.98
3.30	3.43	1.37	2.07	8.64	3.97	4.08	2.98
2.17	1.75	5.98	3.22	1.01	1.33	3.96	4.08
7.49	5.14	5.60	3.23	7.37	3.96	3.74	3.29
5.21	5.08	16.28	10.38	7.80	4.35	7.36	4.64